FREQUENCY SCALING FOR COMPUTER-AIDED FOURIER ANALYSIS OF MIXER DIODE OPERATION

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ABSTRACT

The operation of a mixer diode can be computed using Fourier analysis, which is contained in certain CAD programs such as SPICE. When the difference frequency is much smaller than the input and local oscillation frequencies, greater accuracy can be achieved by scaling down the frequency ratio and scaling up the values of the reactive components in the equivalent circuit.

Fourier Analysis

The operation of a mixer diode[1] can be analyzed with Fourier analysis. The signal frequency component and the local oscillator component are fed to a non-linear diode and the desired difference frequency output is obtained by calculating the difference frequency Fourier component. For an ideal exponential diode, fed from voltage sources, the difference frequency current through the diode is

$$i_\Delta = \frac{1}{\pi}\int_0^{2\pi} I_s\{\exp[V_i \sin\omega_i t + V_\ell \sin\omega_\ell t)/V_T]\}\cdot \{\sin(\omega_\ell - \omega_i)t\}\, d(\omega t) \qquad (1)$$

where $V_i \sin\omega_i t$ is the input signal, $V_\ell \sin\omega_\ell t$ is the local oscillator signal, I_s is the saturation current of the diode and V_T=q/KT is the thermal voltage. This integral can readily be evaluated with numerical methods using programmable calculators or CAD programs especially those furnished with Fourier Analysis routines such as SPICE[2].

For convenience, one can choose $\omega_\Delta = \omega_\ell - \omega_i$ as the fundamental oscillation frequency. Then the input signal frequency and the local oscillation frequency can be considered as harmonics, and the corresponding currents through the diode can also be found by Fourier analysis.

For example, if the signal frequency is 800MHz, local oscillator is 900 MHz, and difference frequency is 100 MHz, then one can analyze the current waveform over a period of 10 nanoseconds. The Fourier Analysis will yield the d-c component, the fundamental component (which is the desired output), the 8th harmonic (which represents the rf-input signal) and the 9th harmonic (which is the current supplied by the local oscillator).

For low frequencies, the difference frequency output is independent of the input frequencies. The reason is that any nonlinear function of $\sin\omega_i t$ and $\sin\omega_\ell t$ can be expanded into a series including terms with $\sin\omega_i t$, $\sin\omega_\ell t$ and $\sin(\omega_\ell-\omega_i)t$, etc. When these terms are multiplied by $\sin(\omega_\ell-\omega_i)t$ and integrated over a cycle, all cross-product integrals become zero and only the $\sin^2(\omega_\ell-\omega_i)t$ term can yield a Fourier coefficient. For instance, the resultant output difference current in Eq. (1) is the same whether $\omega_i = 2\pi \times 900$ MHz and $\omega_\ell = 2\pi \times 800$ MHz, or $\omega_i = 2\pi \times$ 9GHz and $\omega_\ell = 2\pi \times$ 8.9GHz, because in either case the difference radian frequency is $2\pi \times 100$MHz.

If the input frequency is much higher than the difference frequency, the computation becomes increasingly long and time consuming. For example, if the signal frequency is 360 times the difference frequency, then an increment of 1° in the numerical computation is totally inadequate, because each degree of change in the difference frequency corresponds to a change in the input signal of 360°, which reverts the sine function to its original value. A much smaller increment must therefore be used, and results in lengthy computation. Furthermore, in programs such as SPICE, there is an upper limit to the number of time increments for Fourier analysis. Therefore, when the signal frequency is much larger than the difference frequency, the computed result becomes inaccurate. Thus, we should limit the ratio of the signal frequency to the difference frequency to a low value.

Reactive Components

At high frequencies the parasitic components such as the junction capacitance and series resistance play an increasingly important role. A typical equivalent circuit of a junction diode at high frequencies is shown in Fig. 1.

It consists of a series resistance R_s, a nonlinear junction resistance R_j and a nonlinear junction capacitance C. This capacitance can further be divided into a junction capacitance C_j which varies inversely with the reverse bias and a diffusion capacitance C_D which varies as the forward current. These nonlinear capacitances are usually included in the CAD models. In addition, a stray shunt capacitance C_{sh} (due to the package) may exist across the diode. When a combined voltage of input signal, local oscillation and dc bias V_B is impressed across the diode, the voltage appearing across the barrier can be approximated as

$$V_j = V_B + V_i \sin\omega_i t + V_\ell \sin\omega_\ell t. \qquad (2)$$

The current through the diode capacitance is defined by

$$i_C = \frac{dQ(V)}{dt} = \frac{dQ}{dV_j}\frac{dV_j}{dt} = C\frac{dV_j}{dt} \qquad (3)$$

where

Q(V) = charge stored in the diode, which is a function of voltage
C = dQ/dV_j = diode capacitance.

Also, the current through R_j is

$$i_R = I_s[\exp(V_j/V_T)-1]. \qquad (4)$$

These relations, (2), (3), (4), can be applied to Eq. (1) and readily be solved by a computer program such as SPICE. If the local oscillator frequency and the signal frequency are of the same order as the difference frequency, we can substitute the values of capacitance, resistance and frequency directly into the program to obtain the difference frequency output current.

Frequency Scaling

When the actual signal frequency is much larger than the difference frequency as is often the case in microwave applications one cannot use direct substitution for reasons mentioned in the first section. We have developed a frequency scaling technique to tackle this situation.

Combining (2) and (3), one can write the current through the diode capacitance as

$$i_C = C\frac{dv_j}{dt} = C(V_\ell \omega_\ell \cos\omega_\ell t + V_i \omega_i \cos\omega_i t) \quad (5)$$

When the difference frequency ω_Δ is small, then

$$\omega_\ell \approx \omega_i \approx n\omega_\Delta \quad (6)$$

where n is the input signal frequency to difference frequency ratio.

Eq. (5) can now be approximated as

$$i_C = C\frac{dv_j}{dt} = Cn(V_\ell \omega_\Delta \cos\omega_\ell t + V_i \omega_\Delta \cos\omega_i t) \quad (7)$$

Since for computer aided Fourier analysis we should not use a large ratio of ω_i/ω_Δ, let us choose an input frequency $\omega_s = m\omega_\Delta$ in the CAD program, where m is a number less than the maximum number of Fourier components that can be generated by the program. (In SPICE, m should not be larger than 9). Eq. (7) can now be rewritten in terms of the SPICE frequency ω_s

$$i_C \approx C\frac{n}{m}(V_\ell \omega_s \cos\omega_\ell t + V_i \ \omega_s \cos\omega_i t)$$
$$= C_{eq}(V_\ell \omega_s \cos\omega_\ell t + V_i \omega_s \cos\omega_i t) \quad (8)$$

In using the SPICE program, we can choose ω_s as the input frequency and $\omega_s + \omega_\Delta$ as the local oscillator frequency, provided the capacitance is scaled to $C_{eq} = (n/m)C$. Since the signal frequency for the computer-aided program is a small multiple m of the difference frequency, the accuracy of the analysis is high. It should be reiterated that when the current expressed in Eq. (8) is analyzed for the difference frequency current Fourier components, the values of ω_ℓ and ω_i are immaterial so long as $\omega_\Delta = \omega_\ell - \omega_i$.

Example

This technique can be used to analyze the mixing operation of a diode at 8GHz with 100MHz IF output. The difference frequency (IF) is chosen as the fundamental frequency for the CAD. Since SPICE can generate only ten Fourier components (0-9) the 8th component (800MHz) may be chosen to represent the input signal and the 9th component (900MHz) to represent the local oscillator signal, for SPICE. The scale factor for this example is n/m = 80/8 = 10. Therefore if we multiply C by 10, the Fourier Analysis should be valid.

The measured parameters of a silicon diode are as follows:

Junction capacitance at zero bias, C_{j0} = 0.8 pF
Saturation current, I_s = 0.91 nA
Transit time, TT = 10 ns (derived from C_D)
Built-in potential, ϕ = 0.79 V
Series resistance, R_s = 33 ohms
Shunt capacitance, C_{sh} = 0.25 pF

This diode is analyzed in a circuit shown in Fig. 2. The input signal, V_{in}, and the local oscillator signal V_{LO}, are impressed in parallel across the mixer diode through two series resonant LC circuits. The dc bias V_B is also impressed across the mixer diode through an rf choke L_{ch}. The output signal is derived from a local resistance R_L through another series resonant circuit, L_L and C_L.

It should be noted that the applied voltages at the sources V_{in} and V_{LO} differ from the voltages appearing across the diode due to the voltage drop in the source impedances. However since these impedances are in series resonance with the source frequency, the wave shape of the voltage appearing across the diode is unchanged.

The equivalent circuit of the diode is shown inside the dotted outline. The parameters of the diode are listed in the MODEL card in SPICE listing with the exception of R_s. This series resistance is taken outside the diode model for the convenience of sensing the different currents, because all the currents, (i.e. input signal, local oscillator, dc and difference frequency signal) must flow through this resistor, if current through C_{sh} is negligible. (If in doubt, one can sense the output voltage).

Fig. 3 shows the computer analysis of this diode with input signal at 800MHz, local oscillator signal at 900MHz and the difference output signal at 100MHz. The fundamental Fourier Component of the voltage across R_D is the desired difference frequency signal. The current that produces this voltage should flow only through R_L to produce an output voltage. The 8th and 9th harmonic components represent the input and local oscillator currents respectively. Such information can be used to calculate the input impedance and power.

When this diode is operated at 10 times the input signal frequency, we scale the frequency by multiplying the capacitances C_{j0}, C_{sh} and the transit time TT by 10 (as indicated by underlines in Fig. 3 and Fig. 4). The computer printout is shown in Fig. 4. Note that the output of the fundamental frequency, which is the desired difference frequency is larger at 800MHz operation than at 8GHz as expected.

References

[1] D.C. Surana and J. G. Gardiner, "Multiple Fourier Series Analysis for Mixers and Modulators," Proc. IEEE, 59, pp. 1627 - 1628, Nov. 1971.

[2] L. Nagel, "SPICE 2: A Computer Program to Simulate Semiconductor Circuits," ERL, University of California, Berkeley, Ca., ERL-M520, May, 1975.

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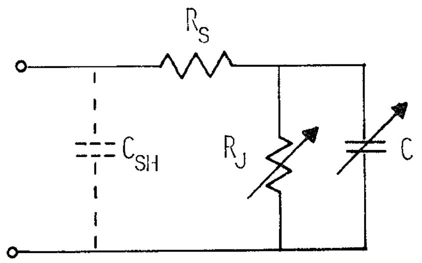

Figure 1. Diode Equivalent Model.

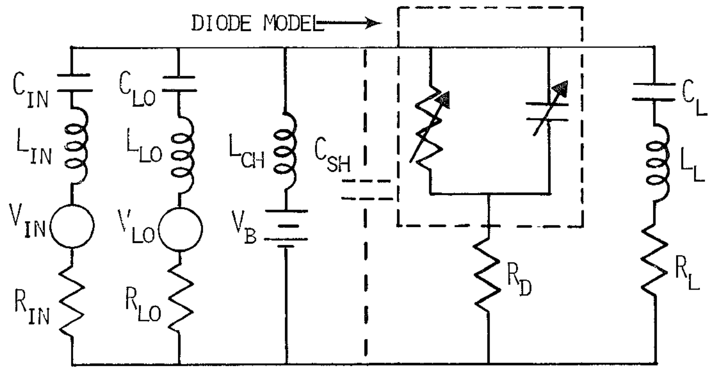

Figure 2. Mixer Circuit.

```
 *RF INPUT SIGNAL (NOT SCALED)
RIN 0 1 400 OHMS
VIN 1 2 SIN 0 0.001 800MEGHZ
LIN 2 3 0.00796MH
 *LOCAL OSCILLATION
RLO  0 5 400 OHMS
VLO  5 6 SIN 0 0.01 900MEGHZ
LLO  6 7 0.00707MH
CLO  7 4 0.00442PF
 *BIAS
VB 8 0 DC 0.5
LCH 8 4 5MH
 *DIODE BRANCH
RD 0 9 33 OHMS
CS 4 0 0.25PF
D 4 9 DIODE
 *LOAD BRANCH
RL 0 10 50 OHMS
LL 10 11 0.01MH
CL 11 4 0.25PF
.NP
.MODEL DIODE SBD RS=0 TT=10NS CJO=.8PF IS=91.0E-14 PHI=.79
.OUTPUT VOUT 9 0
.TRANS .10NS FOUR VOUT 100MEGHZ
.END
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE OF VOUT

DC COMPONENT = 5.985-03

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000+08	2.232-06	1.000000	73.082	.000
2	2.000+08	2.794-06	1.251534	53.623	-19.459
3	3.000+08	3.601-06	1.613135	42.278	-30.804
4	4.000+08	4.805-06	2.152592	34.407	-38.675
5	5.000+08	6.663-06	2.984848	28.823	-44.260
6	6.000+08	9.897.06	4.433889	24.705	-48.378
7	7.000+08	1.730-05	7.752198	21.516	-51.567
8	8.000+08	3.177-05	14.233068	29.899	-43.183
9	9.000+08	9.315-05	41.729480	121.898	48.816

Figure 3. SPICE Output of Fourier Components of Mixer Operating at 800 MHz.

```
 * RF INPUT SIGNAL (SCALED FOR  8 GHZ)
RIN  0 1 400 OHMS
VIN  1 2 SIN 0 0.001 800MEGHZ
LIN  2 3 0.00796MH
CIN  3 4 0.00497PF
 *LOCAL OSCILLATION
RLO  0 5 400 OHMS
VLO  5 6 SIN 0 0.01 900MEGHZ
LLO  6 7 0.00707MH
CLO  7 4 0.00442PF
 *BIAS
VB 8 0 DC 0.5
LCH 8 4 5MH
 *DIODE BRANCH
RD 0 9 33 OHMS
CSH 4 0 2.5PF
DB 4 9 DIODE
 *LOAD BRANCH
RL 0 10 50 OHMS
LL 10 11 0.01MH
CL 11 4 0.25PF
.NP
.MODEL DIODE SBD RS=0 TT=100NS CJO=8PF IS=91.0E-14 N=1 PHT=.79
.OUTPUT VOUT 9 0
.TRANS .10NS 10NS FOUR VOUT 100MEGHZ
.END
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE OF VOUT

DC COMPONENT = 5.983-03

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000+08	9.882-07	1.000000	-54.855	.000
2	2.000+08	1.727-06	1.747921	-28.981	25.874
3	3.000+08	2.589-06	2.619628	-19.368	35.487
4	4.000+08	3.766-06	3.810470	-13.877	40.978
5	5.000+08	5.509-06	5.574881	-10.304	44.551
6	6.000+08	8.489-06	8.590422	- 7.778	47.076
7	7.000+08	1.527-05	15.453282	- 5.836	49.019
8	8.000+08	2.792-05	28.247864	4.940	59.795
9	9.000+08	8.228-05	83.256449	100.289	155.144

Figure 4. SPICE Output of Fourier Components of Mixer Operation at 8 GHz.